

# Prediction of Clean-Bed Head Loss in Crumb Rubber Filters

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**Abstract:** Crumb rubber media have been beneficially invented for ballast water and tertiary wastewater treatment. There is a critical need for precise prediction of clean-bed head loss for engineering design purposes. Pilot crumb rubber filters were tested to study their clean-bed head loss under the influences of three design and operational parameters (media size, media depth, and filtration rate). Data from the filtration tests were used to evaluate the application of the Kozeny and Ergun equations in crumb rubber filters and to develop a statistical model for examination of the effects of each parameter and for clean-bed head loss prediction. Results showed that both the Kozeny and Ergun equations had limitations for crumb rubber filters, especially when the data of compressed media depth were unavailable from the filtration tests. The statistical model developed by the multiplicative power-law relationship was proved to be valid, and it could be used to predict clean-bed head loss in crumb rubber filters.

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**Author keywords:** Crumb rubber; Clean-bed head loss; Filtration; Ergun equation; Kozeny equation.

## Introduction

Approximately 290 million scrap tires are generated each year in United States, and their stockpiles cause health and environmental concerns by presenting a potential fire hazard and providing breeding ground for vectors of disease (U.S. EPA 2006). Various recycling technologies have been recommended for conservation of natural resources and minimization of environmental impacts of these tires (Sunthonpagasit and Hickman 2003). The use of crumb rubber, a scrap-tire-derived material, as a filter medium is an innovative technology that has been investigated as a potential “green engineering solution” for wastewater treatment and disposal of scrap tires. As a compressible material, crumb rubber forms an ideal porosity gradient in filters because the top layer of the media is least compressed while the bottom layer is most compressed. Crumb rubber filters favors in-depth filtration and allows longer filtration time and higher filtration rate, which substantially increases the filtration efficiency (Graf and Xie 2000; Xie et al. 2001). These filters’ light weight, compact size, and

capability to remove turbidity, phytoplankton, and zooplankton also allow them to be beneficially used in ballast water treatment (Tang et al. 2006a, 2009).

To design a crumb rubber filter, it is important to understand the effects of design and operational parameters on filter performances. Clean-bed head loss is one important filter performance parameter because sufficient water head must be provided to accommodate the increase of head loss resulting from the accumulation of particulates in filter media (Cleasby and Logsdon 1999). For decades, the Kozeny equation (Kozeny 1927a,b; Fair and Hatch 1933; Carman 1937; Fair et al. 1968) and the Ergun equation (Ergun and Orning 1949; Ergun 1952) are most accepted among numerous models for prediction of clean-bed head loss (Trussell and Chang 1999). Their expressions are shown in Eqs. (1) and (2). The two equations, however, were developed for conventional rigid granular media filters. Their applicability in crumb rubber filters was not clear because of the compression of crumb rubber media, which changed the filter configurations and porosity during the filtration process, especially at the higher filtration rates that crumb rubber filters are designed to be operated.

The objective of this research was to evaluate the Kozeny and Ergun equations for clean-bed head loss prediction in crumb rubber filters. Data from the filtration tests were also used to investigate the effects of three design and operational parameters on head loss and to develop a statistical model for improved clean-bed head loss prediction for such filters.

## Materials and Methods

### Filter Setup

The filter study was carried out in the Kappe Environmental Engineering Laboratory at The Pennsylvania State University Wastewater Treatment Plant, University Park, Pennsylvania, U.S.A. A pilot filter column (Fig. 1) was constructed with two 15.2-cm-diameter transparent PVC pipes. A water pump was used to sup-

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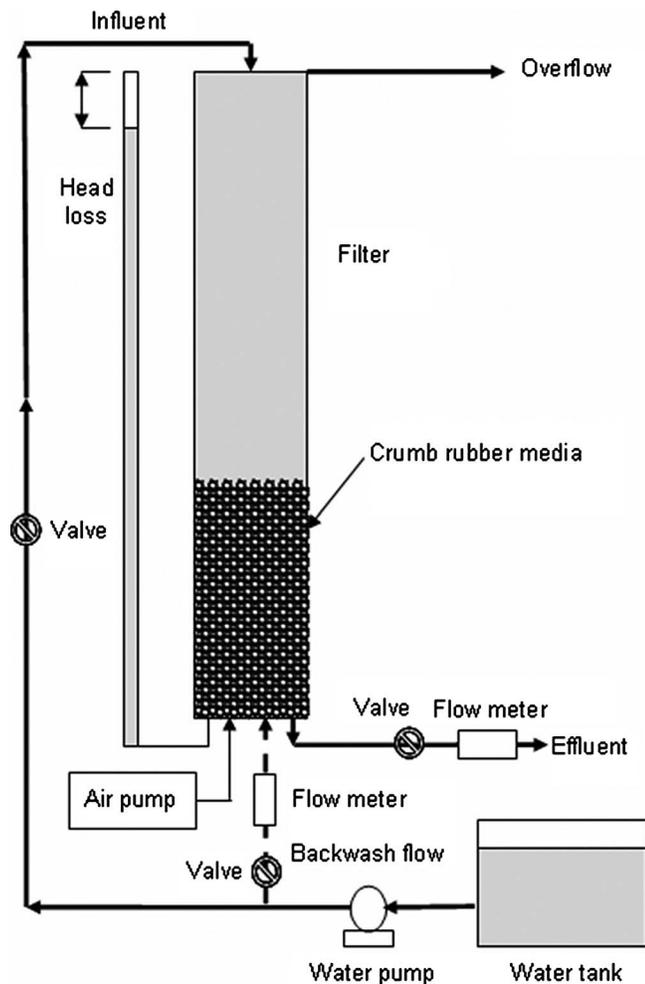


Fig. 1. Experimental setup of the crumb rubber filter

ply the influent and backwash flow from a water storage tank. An air pump was connected to the bottom of the filter column for air scour before water backwashing. The filtration rate was controlled by a flow meter installed in the filter column outlet pipe. Influent water head was kept constant by the use of an overflow at 3.29 m. The head loss through the filter media was measured using the difference between the water level in the filter column and the water level in the glass tube connected to the bottom of the filter column.

### Filter Media

The crumb rubber media size was determined by sieve analysis using ASTM Standard Test C136-01, Sieve analysis of Fine and Coarse Aggregates (ASTM 1993). The effective sizes were 0.66, 1.20, and 1.90 mm. Their uniformity coefficients were determined to be 1.39, 1.53, and 1.28, respectively, and the packed porosities were 0.62, 0.58, and 0.54 for 0.66-, 1.20-, and 1.90-mm crumb rubber media, respectively. The density of all crumb rubber media was 1,130 kg/m<sup>3</sup>.

### Design and Operational Conditions

The study used a mixed-level factorial design to analyze the effects of factors and possible interactions. Three levels of crumb rubber media sizes and media depths and 19 filtration rates were

investigated. The filter column was loaded with 0.66-, 1.20-, and 1.90-mm crumb rubber media. For each size, the filter column was loaded to depths of 0.6, 0.9, and 1.2 m. Before each filter run, the filter was backwashed by air scour and then water flowed at the rate of 29.3 m<sup>3</sup>/m<sup>2</sup> h. For each filter configuration, the filter was operated at 19 filtration rates from 0 to 73.3 m<sup>3</sup>/m<sup>2</sup> h (0, 4.9, 9.8, 12.2, 14.7, 19.6, 24.4, 29.3, 34.2, 36.7, 39.1, 44.0, 48.9, 53.8, 58.7, 61.1, 63.6, 68.4, and 73.3 m<sup>3</sup>/m<sup>2</sup> h). The maximum filtration rate was selected based on results from Tang et al. (2006a) where a maximum flow rate of 73.3 m<sup>3</sup>/m<sup>2</sup> h was used for ballast water treatment by crumb rubber filtration. Head loss was measured after the media depth was stable. Porosity was determined by the measurement of the dry weight of the media initially loaded to the filter columns and the media depth (Trussell et al. 1999). Sphericity value for each medium was determined by empirical fitting of data from the filtration tests (Crittenden et al. 2005).

### Statistical Modeling

A total of 171 filtration data sets were collected (3 media sizes × 3 media depths × 19 runs). Based on statistical requirements, 70% of data sets were chosen to initiate the regression and the remaining 30% were used to validate the corresponding regression results. Regressions were performed using a least-squares criterion and assuming that the residuals were normally distributed and independent and had constant variance. These assumptions were checked after the model had been fitted. Sigma Plot 10 (Systat Software, Inc., Chicago) was used to fit the data for media sphericity and to conduct the statistical multiple nonlinear regression for developing a statistical model.

### Regression Verification

For each attempted fit of sphericity to actual head loss data, the hypothesis tested was that the derived sphericity for the equation was significantly different from zero. By examining the *p*-value of each coefficient, the null hypothesis can be rejected if the *p*-value was less than 0.05.

Analysis of means was performed using two-sample t-test to compare model coefficients. For sphericity estimates, the hypothesis tested was that the derived value from the Kozeny equation was significantly different from the one from the Ergun equation. Pooled variance or separate variance t-test was used to compute the t-statistic depending on the difference of variances (Ott and Longnecker 2000), with  $\alpha=0.05$  in this test.

The coefficient of determination ( $R^2$ ) was another criterion used to verify these regressions. It represents the proportion of the total variability in the dependent variables explained by the regression equation accounts (Berthouex and Brown 1994). An  $R^2$  of 1.0 indicates that the equation addresses all the variability of dependent variables, and it generally indicates a strong relation if  $R^2$  is large. However, it does not guarantee a statistically valid equation since high  $R^2$  can occur with insignificant coefficient if only a few data observations are available. Regressions in this study were validated using the following approaches: (1) comparing the regression  $R^2$  and verification  $R^2$ ; (2) examining the *p*-value of model coefficients; and (3) examining the residuals of the resultant regression graphically. In addition, the standard error of each coefficient, which was computed from the variance of the predicted values, was used as a measure of the variability.

Graphical analyses of regressions were performed by examining the following requirements according to Draper and Smith

**Table 1.** Sphericity Estimates of Crumb Rubber Media for the Kozeny and Ergun Equations

Equation	Media size (mm)	Sphericity estimate	Standard error	<i>p</i> -value	Lower 95(%)	Upper 95(%)	Regression		
							<i>R</i> <sup>2</sup>	Verification <i>R</i> <sup>2</sup>	Normality test <sup>a</sup>
Kozeny	0.66	0.672	0.006	<0.0001	0.660	0.684	0.973	0.929	F
	1.20	0.637	0.009	<0.0001	0.655	0.932	0.932	0.834	P
	1.90	0.620	0.009	<0.0001	0.638	0.934	0.934	0.822	F
Ergun	0.66	0.710	0.004	<0.0001	0.701	0.719	0.991	0.992	F
	1.20	0.749	0.009	<0.0001	0.730	0.768	0.965	0.930	P
	1.90	0.794	0.007	<0.0001	0.781	0.808	0.987	0.917	F

<sup>a</sup>F=Fail; P=Pass.

(1981): (1) residuals are independent; (2) residuals have zero mean; (3) residuals have constant variance; and (4) residuals follow a normal distribution. The purpose was to determine if the assumption of the regression error being independent and normally distributed was valid. Verification of the assumption of independent residuals was performed by graphically plotting the residuals against its variables (filtration rate, media depth, and media size) and observed values. For this assumption to be valid, the residuals should be randomly distributed in these four plots around a zero line. Verification of the assumption of normal distribution of residuals was performed using a normal probability plot. Residuals that fall along a straight line in the plot and fall into the 95% confidence interval are considered to be normally distributed. If residuals fail to pass any one of these tests, then the regression is not valid for the data.

ANOVA was performed to check constant variances for some instances of the models if they passed the independence and normality tests. The hypothesis tested was that the variances were significantly different from each other. The Levene test was performed with the assistance of MINITAB 15 (Minitab, Inc., State College, Pa). If the *p*-value was less than 0.1, the null hypothesis can be rejected and the alternative one can be concluded that the variances were not equal. MINITAB 15 was also used to create and analyze factorial design to determine the actual effects of factors.

## Results and Discussion

### Media Sphericity

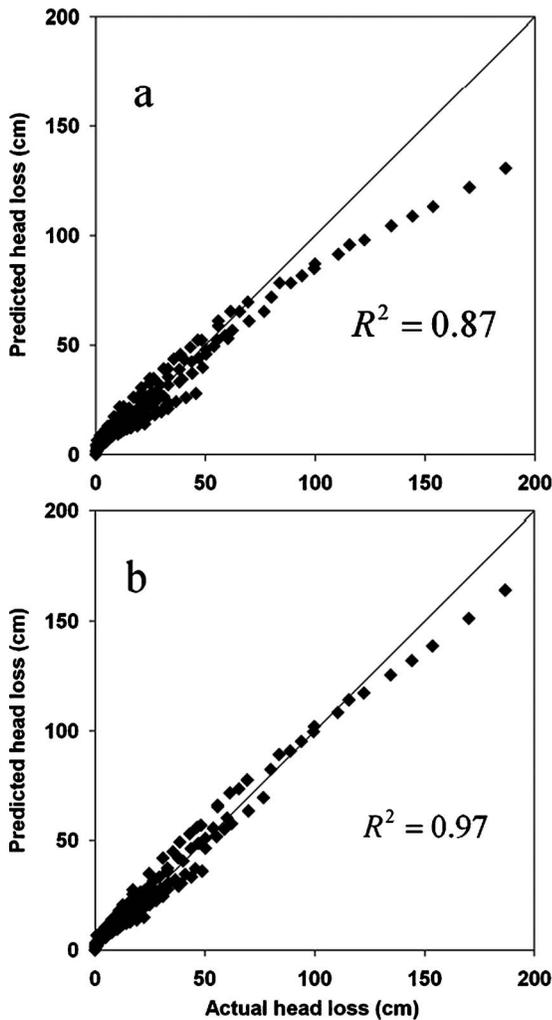
Sphericity, a physical parameter that defines the roundness of the media shape, has to be provided if the Kozeny and Ergun equations are used. For crumb rubber media, their values were estimated by empirical fitting of the actual head loss data to the predicted head loss data computed by the two equations using the compressed media depth and porosity. Two initial assumptions were made before the regressions: (1) sphericity values for different media sizes were the same and (2) sphericity values at different operating conditions were the same. Based on the assumptions, regressions were first performed using 70% of all data sets without differentiating media sizes. However, the residuals of both equations could not pass the normality test and therefore made the regressions invalid, indicating that the assumptions could be partially wrong. The first assumption was then modified as sphericity values for different media sizes were different. With the modified assumptions, regressions were individually performed using 70% of data sets of each media size, and the results were summarized in Table 1. The Kozeny equation gave a sphericities of 0.67, 0.64, and 0.62, while the Ergun equation gave sphericities of 0.71, 0.75, and 0.79 for the 0.66-, 1.20-, and

1.90-mm crumb rubber media, respectively. All the *p*-values were less than 0.0001, showing that these coefficients were significant. Results of two-sample t-test indicated that a decreasing trend did not exist for the estimates from the Kozeny equation while there was an increasing trend for the estimates from the Ergun equation. In addition, it was statistically proved that the Ergun equations gave a higher sphericity estimate than the Kozeny equation. The 95% confidence intervals gave a range of variability for those estimates. It has to be noted that the regressions for 0.66- and 1.90-mm media failed the normality test, showing that the second assumption could be incorrect for the two sizes. That is, for the 1.20-mm crumb rubber media, it was acceptable to assume that sphericity did not change due to compression but for the other two sizes; this assumption might be incorrect. However, these estimates were still applied in the evaluation process of the two equations because these estimates provided the highest regression *R*<sup>2</sup> and verification *R*<sup>2</sup>, which addressed the most variability of the dependent variables.

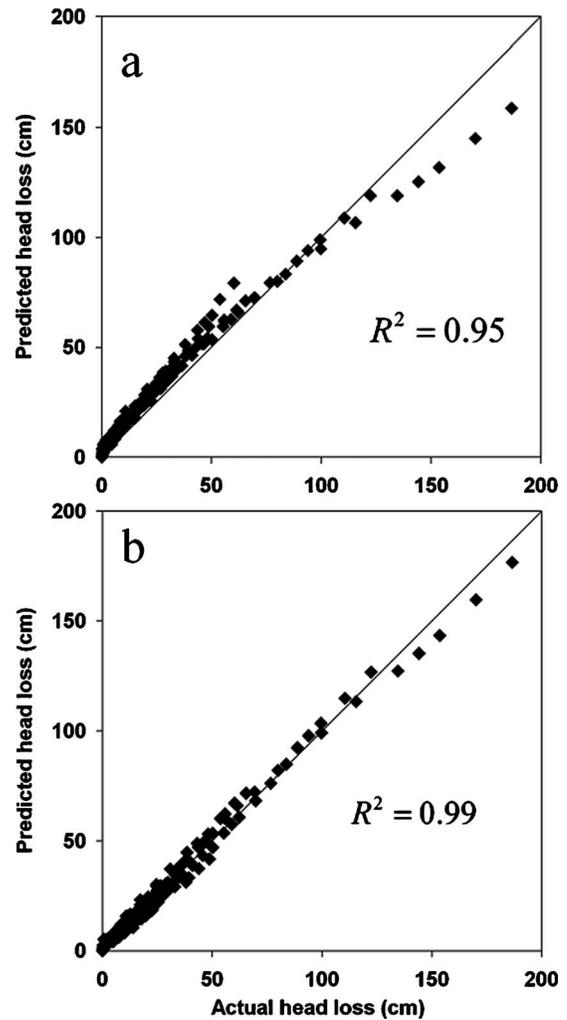
### Prediction by the Kozeny and Ergun Equations

Figs. 2 and 3 compared the actual head loss with the predicted head loss by the Kozeny and Ergun equations using the initial and compressed media depth, respectively, to evaluate the applicability of the two equations in crumb rubber filters. The 45° line in the figure depicted the hypothetical head loss estimates that were precisely equal to the actual values (Tang et al. 2006b). Coefficient of determination was calculated between the predicted head versus actual head loss and the 45° line. As shown in Fig. 2(a), with an *R*<sup>2</sup> value of 0.87, the Kozeny equation underestimated head loss, especially at high filtration rates for each filter medium configuration. This implied that the Kozeny equation has limitations for crumb rubber filters if compressed media depth data were not available to compensate the media compression. The underestimation was likely due to the decreased porosity resulted from the media compression. Fig. 3(a) shows that the Ergun equation gave an *R*<sup>2</sup> of 0.95, which was higher than that of the Kozeny equation using the same data sets, although it still did not account for all the variability. This suggested that the inert head loss included by the Ergun equation could improve the performance of head loss prediction for crumb rubber filters. The variability which was not addressed still could come from the head loss increase due to media compression and the decrease of porosity.

When the data sets of compressed media depth from the filtration tests were applied to both equations to reflect media compression, it was found [Fig. 2(b)] that the Kozeny equation could give better predicted values with an *R*<sup>2</sup> value of 0.97, which indicated that more variability could be addressed by the Kozeny equation in this case. However, it was also noticed that some predicted head losses at high filtration rates were underestimated,



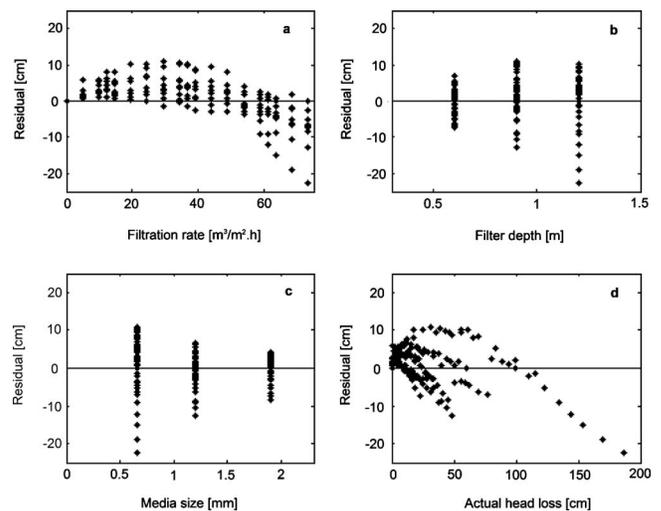
**Fig. 2.** Prediction of clean-bed head loss by the Kozeny equation: (a) prediction using the initial media depth; (b) prediction using the compressed media depth



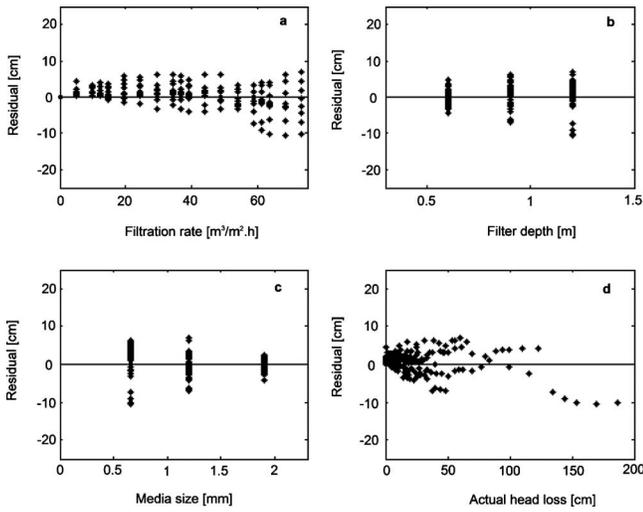
**Fig. 3.** Prediction of clean-bed head loss by the Ergun equation: (a) prediction using the initial media depth; (b) prediction using the compressed media depth

which indicate that the equation had limitation when the filtration rate was high at each filter medium configuration. For the data shown in Fig. 3(b), the  $R^2$  value of the Ergun equation was 0.99. Comparing with the Kozeny equation, the Ergun equation had higher  $R^2$  and showed a better fit to the actual head loss. In addition, the Ergun equation did not underestimate head loss at high filtration rates in the way the Kozeny equation did. In addition, the inclusion of media compression using the Ergun equation was able to account for the variability that could not be addressed when the crumb rubber filters were treated as rigid granular media filters as shown in Fig. 3(a).

Figs. 4 and 5 show the residuals of both equations against three variables and actual head loss. It was found that the residuals were independent against filter depth and media size because they were almost centered at the zero line [Figs. 4(b and c); Figs. 5(b and c)]. However, they were dependent on filtration rate and actual head loss. As filtration rate or actual head loss increased, residuals went from positive to negative values [Figs. 4(a and d); Figs. 5(a and d)]. The results of residual analysis were evaluated against the criterion of independent residuals for a good model. Therefore, although the  $R^2$  of the Ergun equation was relatively



**Fig. 4.** Residual analysis of the Kozeny equation using the compressed media depth



**Fig. 5.** Residual analysis of the Ergun equation using the compressed media depth

higher than the Kozeny equation, both equations had limitations and the derived sphericity estimates could not describe the real shape of the grains.

### Development of a Statistical Model

From previous discussion, it can be summarized that it is difficult to obtain the sphericity of crumb rubber media because this parameter is not constant as it is in conventional rigid granular media. The empirical fitting approach for sphericity has the limitations described above for the crumb rubber media. In addition, both the Kozeny and Ergun equations had deficiencies for the prediction of clean-bed head loss in crumb rubber filters. To maximally compensate for the media compression, which is different from rigid granular media filters, the data of actual compressed media depth had to be used in the two equations. The predictions using the data from the initial media depth could not provide the same degree of comparison to the actual head loss data.

The benefits of developing a statistical model for crumb rubber media are the following: (1) no need to conduct filtration tests in

order to obtain actual compressed media depth and porosity; (2) no need to derive a sphericity estimate since this value varies according to the filtration conditions; and (3) both the Kozeny and Ergun equations have limitations for crumb rubber filters.

The statistical model included several empirical model parameters based on fundamental theoretical considerations to describe the results of pilot experiments (Tobiason and Vigneswaran 1994). It used a multiplicative power-law relationship, which resembled the Kozeny equation. The model, initially expressed in Eq. 3, consisted of the three factors examined in this study as independent variables: filtration rate, media depth, and media size. Porosity is not an independent parameter but instead is a function of media depth; therefore, it was not included. Exponents of each factor and the numerical constant were obtained via regression using 70% of data sets. The initial assumption was that the model coefficients were the same for all media sizes. However, the resultant group of coefficients failed the normality test, indicating the assumption was probably incorrect. The assumption was then modified as follows: the model coefficients were different among media sizes. The model then had a form expressed in Eq. (4).

Regressions then were performed individually using 70% of data sets for each media size based on Eq. (4), and all the three groups of coefficients passed the normality tests. As summarized in Table 2, the  $p$ -values were less than 0.0001, indicating that the coefficients were significant. Standard errors and 95% confidence intervals gave the ranges of the variability. Regression  $R^2$  and verification  $R^2$  of the model were higher than 0.996, which demonstrated that the statistical model had a good fit to actual head losses.

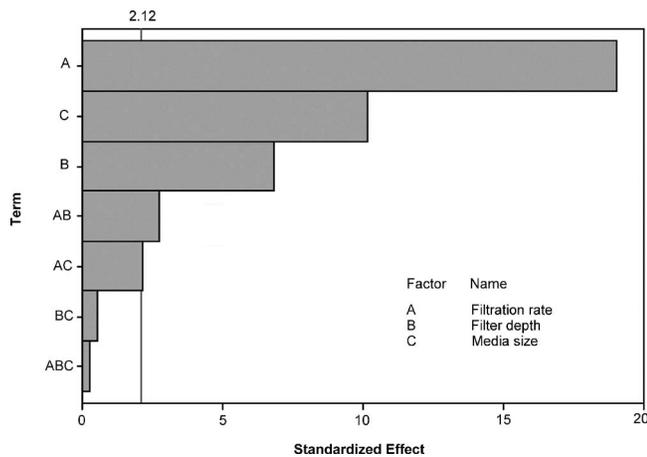
### Effect of Parameters

For rigid granular media filters, the Kozeny and Ergun equations showed that there was a fixed relationship between head loss and any of the three design and operational parameters tested in this study. Because crumb rubber is a compressible media, the exploration of effects of each parameter, assisted by the statistical model, was important to understand how the clean-bed head loss was affected by media size, media depth, filtration rate, and their interactions and whether these effects were different from those seen using the Kozeny and Ergun equations. Factorial calculations were conducted for the above purpose.

**Table 2.** Parameters in the Statistical Model

Media size (mm)	Parameter	Coefficient	Standard error	$p$ -value	Lower 95(%)	Upper 95(%)	Regression $R^2$	Verification $R^2$	Normality Test <sup>a</sup>
All sizes	K	0.0076	0.0017	<0.0001	0.0042	0.0110	0.991	0.990	F
	a	1.55	0.03	<0.0001	1.50	1.61			
	b	1.29	0.03	<0.0001	1.23	1.36			
	c	-1.54	0.03	<0.0001	-1.60	-1.49			
0.66	K	618	54	<0.0001	508	729	0.998	0.998	P
	a	1.55	0.02	<0.0001	1.51	1.59			
	b	1.35	0.02	<0.0001	1.30	1.39			
1.20	K	185	23	<0.0001	138	233	0.996	0.997	P
	a	1.51	0.03	<0.0001	1.45	1.57			
	b	0.97	0.03	<0.0001	0.91	1.03			
1.90	K	342	49	<0.0001	242	441	0.996	0.996	P
	a	1.75	0.03	<0.0001	1.68	1.82			
	b	1.21	0.03	<0.0001	1.14	1.28			

<sup>a</sup>F=Fail, P=Pass.

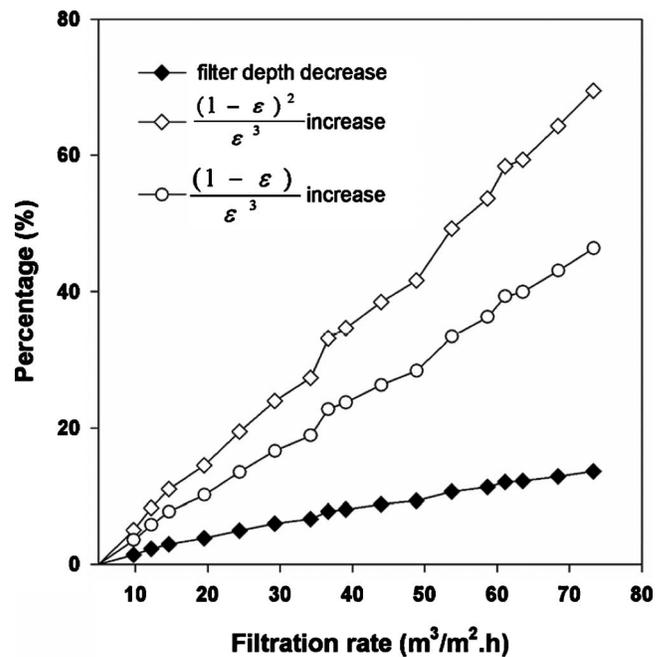


**Fig. 6.** Pareto chart of the standardized effects showing the results of medium and high level factorial calculation. (Response is observed head loss in centimeters. Significance level=0.05.)

Although mixed-level factorial design was conducted in this experiment, only the result of medium and high level factorial calculation is shown in Fig. 6. These calculations are more meaningful since crumb rubber filters are usually designed to be operated at medium and high levels of filtration rates. The threshold value was determined to be 2.12 by the method of Lenth (1989) with the significance level of 0.05 and was denoted by the vertical line in the Pareto chart of the standardized effects (Fig. 6). Effects of filtration rate, media size, media depth, interaction of filtration rate, and media depth and interaction of filtration rate and media size were judged as significant because their  $p$ -values are less than 0.05. The absolute values of their standardized effects were 19.06, 10.17, 6.82, 2.77, and 2.13, respectively. The interaction of filtration rate and media depth could be explained by the compression because the increase of filtration rate correspondingly decreases media depth and therefore confounds head loss. The interaction of filtration rate and media size, although very small, was just above the threshold value. The interaction was likely due to the change of sphericity during compression because the assumption of equal sphericity at different filtration conditions were proved to be incorrect for some media sizes. The Kozeny and Ergun equations could not address these distinctive effects of crumb rubber media. The statistical model developed by the multiplicative power-law relationship could be used to analyze these effects on clean-bed head loss in crumb rubber filters. Interaction of filter depth and media size and interaction of all three factors were statistically insignificant; therefore, they were not included in the following discussion.

#### Effect of Filtration Rate

The exponents of filtration rate, shown as 1.55, 1.51, and 1.75 for 0.66-, 1.20-, and 1.90-mm crumb rubber media, respectively, in the statistical model were all higher than the 1 used in the Kozeny equation. This could explain the underestimation of the Kozeny equation because the filtration rate in crumb rubber filters had a larger impact than it would in other conventional granular media filters. This caused the actual head loss to increase faster when the filtration rate was high. The faster increase was due to the rapid development of inert head loss that could not be addressed by a linear relationship between head loss and filtration rate.



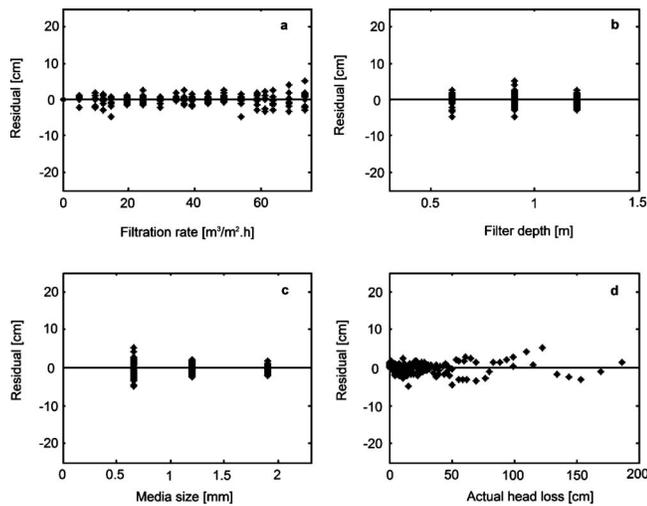
**Fig. 7.** Impact of media compression for the filter configuration of 0.66-mm crumb rubber media and 0.6-m media depth

#### Effect of Interaction between Filtration Rate and Media Depth

The larger exponent of filtration rate was also believed to be caused by the interaction between filtration rate and media depth due to compression of the filter media, which is shown in Fig. 7. The filter depth was decreased as the filtration rate increased. It was found that the crumb rubber filter with a media size of 0.66 mm and filter depth of 0.6 m was compressed by 14% as the filtration rate increased from 0 to 73.3 m³/m² h. However, for conventional granular filters, the filter depth and filtration rate were independent parameters. In crumb rubber filters, the phenomenon could not be addressed by head loss models developed for rigid granular media filters.

#### Effect of Media Depth

The exponents of media depth, shown as 1.35, 0.97, and 1.21 for 0.66-, 1.20-, and 1.90-mm crumb rubber media in the statistical model showed the influence of media compression on clean-bed head loss. The exponent was 1 in both the Kozeny and Ergun equations. The effect of media compression in crumb rubber filters was complicated according to the head loss theories. The compression decreased the media depth, which could decrease head loss, but it also decreased the porosity at the same time, which could increase head loss. Fig. 7 shows the media depth and porosity term change. For a crumb rubber filter loaded with 0.66-mm crumb rubber media to a depth of 0.6 m, as the filtration rate gradually increased from 0 to 73.3 m³/m² h, the media depth was steadily decreased by 13.6%. However, the decrease in media depth did not cause a corresponding decrease in its exponent, which was 1 as shown in the Kozeny and Ergun equations. In fact, the exponent of media depth was increased to 1.35 because the porosity term in the Kozeny equation was increased by 69.5% due to the compression. The porosity term in the second part of the Ergun equation did not increase as much as the term in the Kozeny equation, which only increased by 46.4%. It still played an important role when the filtration rate was high and the inert



**Fig. 8.** Residual analysis of the statistical model using the initial media depth

head loss was dominant. Therefore, the exponent of media depth implied the overall influence of media depth decrease and porosity term increase in crumb rubber filters.

#### Effect of Media Size

The exponents of media size, shown as  $-1.54$  in the statistical model, which was obtained by no differentiation of media sizes, were between  $-2$  in the Kozeny equation and  $-1$  in the second term of Ergun equation, indicating that the Ergun equation might be able to address the effect of this factor while the Kozeny equation could not.

#### Effect of Interaction between Filtration Rate and Media Size

Filtration rate affected crumb rubber media also by changing the particles' sphericity. The assumption of equal sphericity was unacceptable for some media sizes when their sphericity estimates were verified in the study. The two conventional equations could not address this problem because they both assumed that sphericity will not change. However, it was not necessarily true for crumb rubber media. Compression could change the shape of the media especially when the filters were operated at higher filtration rates, which correspondingly exerted higher pressure on compressible media.

#### Prediction by the Statistical Model

Because the statistical model was developed using actual head loss, initial media depth, filtration rate, and media size, it included the effect of media compression. It also addressed the problems encountered by the Kozeny and Ergun equations, such as that tests must be conducted at all filtration rates to obtain the data for compressed media depth and porosity, interactions issues among parameters, and hard-to-determine sphericity issues.

Fig. 8 shows the residuals of the statistical model against three variables and actual head loss. It was found that the statistical model did not underestimate head loss at high filtration rates in the way the Kozeny and Ergun equations did. Also, the residuals were independent against all variables since they were all centered at the zero line. In addition, when the hypothesis of equal variances of residuals against actual head loss was tested, the  $p$ -value for Levene's test was 0.122. Comparing to the confidence level of 0.1 (the default choice for rejecting the hypothesis of

equal variances), the  $p$ -value was higher, and therefore, no conclusion could be made that they were not equal. Because normality, independence, and constant variances all met the verification requirement, the statistical model was valid for clean-bed head loss prediction for crumb rubber filters.

## Conclusions

1. Both the Kozeny and Ergun equations had limitations in clean-bed head loss prediction in crumb rubber filters, especially when the data for actual compressed media depth and porosity were unavailable to compensate for the media compression.
2. The effects of filtration rate, media depth, media size, and their interactions in crumb rubber filters were different from conventional rigid granular media filters due to the media compression.
3. The statistical model developed by the multiplicative power-law relationship is valid and can be used in predicting clean-bed head loss in crumb rubber filters.

## Acknowledgments

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## Notation

The following symbols are used in this paper:

- $a$  = exponent of filtration rate;
- $b$  = exponent of media depth;
- $c$  = exponent of media size;
- $d_{eq}$  = equivalent size of filter media;
- $g$  = acceleration of gravity;
- $h$  = head loss;
- $K$  = dimensionless constant for the statistic model;
- $k$  = dimensionless Kozeny constant commonly found close to 5 under most filtration conditions (Fair et al. 1968);
- $k_2$  = dimensionless constant found to be 0.48 for crushed media (Ergun 1952);
- $L$  = media depth;
- $V$  = filtration rate;
- $\varepsilon$  = porosity of filter media;
- $\mu$  = absolute viscosity of fluid;
- $\rho$  = mass density of fluid; and
- $\psi$  = sphericity.

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